Average Return, Return Volatility, & Return Compounding

Issue

Researchers often evaluate stock selection criteria by ranking raw factor values into uniform groups such as quintiles or deciles. Factor strength is typically quantified by computing average returns for each rank cohort over time. The larger the average return spread between favorably ranked and unfavorably ranked stocks, the stronger and more desirable a factor is considered to be. However, average returns across rank cohorts don't tell the whole story.

The variability behind those average returns and the volatility of average returns through time is often ignored in factor analysis. This is puzzling since return volatility is a commonsense risk measure reflecting investor preference for strategies with consistent returns over those with variable returns. How much additional factor performance perspective do return volatility statistics provide? Can factors with weak return prediction actually be useful if they're strongly correlated with volatility? How do average returns, return volatility, and compound (i.e., geometric) returns interact?

Research Approach

To answer these questions, we focused our research attention on three investment factors more commonly used to predict risk than to predict returns.

Table 1: Investment Factors	
Factor	**Definition**
Price Volatility	12-month standard deviation of returns
Beta	60-month covariance vs S&P 500
Market Cap	Stock price x shares outstanding

Each month from November 2001 – April 2022, we calculated factor values for all MSCI U.S. IMI members (approximately the largest 2300 market cap stocks at each point in time; REITs excluded) and ranked stocks into quintile groups. We computed the subsequent mean returns and standard deviation of returns for each rank cohort, and then averaged those metrics over the entire test period. We also quantified risk-adjusted returns by computing the average monthly alpha for each rank cohort. Finally, we derived the geometric return of each factor cohort by compounding returns into a wealth index.

Results

Let us first provide some background before examining actual test results. The cost of volatile returns is mathematical, not just psychological. For example, most investors have heard comments such as "a 25% loss requires a 33% gain to get back to breakeven". But all return variability, not just downside variability, negatively impacts return compounding. This detrimental effect is sometimes called "volatility drag" or "variance drain". A useful formula for *estimating* volatility drag on annual return statistics is:

G = A - ((S^2) / 200)

G is Geometric (or compounded) return
A is Average return
S is Standard deviation of return.

To illustrate, Table 2 shows two stylized strategies with positive four-year returns that differ only by their variability. Note how the lower volatility strategy produces a higher compound return. The geometric return of *any* return series with variability is less than the average return, and the return gap grows as variability increases. Since small differences in geometric return compound to large differences in wealth over time, the volatility drag formula provides an important and often overlooked insight, namely that there are *two* approaches to increasing compound wealth:

- raising average returns
- reducing return volatility

Table 2: Return Volatility Reduces Compounding					
4-Yr Return Sequence	**Avg Return**	**Stdev Return**	**Growth of $1.00**	**Actual Geom Return**	**Estd Geom Return**
8%,12%,8%,12%	10.0%	2.3%	$1.463	9.98%	9.97%
2%,18%,2%,18%	10.0%	9.2%	$1.449	9.71%	9.57%

Let's move on to some actual test results. Table 3 summarizes average returns and return variability across the ranking spectrum for our three investment factors. As one would expect for risk-oriented metrics, factor levels have had little correlation with average returns but strong correlation with return volatility. Since average returns were similar across the quintile ranks, most researchers would conclude that price volatility, beta, and market cap are useless as return prediction factors. End of story?

Table 3: Monthly Return vs Volatility by Quintile						
Factor	**Statistic**	**1**	**2**	**3**	**4**	**5**
PriceVol	AvgRet%	0.94	1.01	0.99	0.90	0.92
Beta	AvgRet%	0.84	1.03	1.13	1.13	0.86
Mktcap	AvgRet%	0.89	0.95	0.94	1.03	0.95
PriceVol	StdvRet%	5.85	7.57	9.27	11.77	16.80
Beta	StdvRet%	9.94	8.77	9.37	10.31	12.60
Mktcap	StdvRet%	7.33	8.88	10.42	12.21	14.75
PriceVol	Alpha%	0.39	0.23	0.07	-0.19	-0.51
Beta	Alpha%	0.23	0.28	0.25	0.06	-0.46
Mktcap	Alpha%	0.17	0.10	-0.02	0.00	-0.26

RIR asserts that ignoring return variability can be a big mistake when factor levels are strongly correlated with return variability like those in Table 3. The alphas in the bottom panel suggest that these risk factors have had positive correlation with subsequent risk-adjusted returns. Some analysts have argued that positive excess returns are unambiguously desirable whereas "you can't eat alpha". Funny, but is this statement true?

Let's look more closely at the Price Volatility factor. Table 3 shows that stocks ranked in quintiles 1 and 5 have had similar average returns but very different return volatility through time. We suspect that many readers would predict that a hypothetical investment in each cohort "portfolio" would deliver similar long-term returns but that the quintile 1 portfolio would provide a smoother ride. That prediction would be right on the volatility outcome but very wrong on the return outcome.

Figure 1 shows the startling difference in growth of $1.00 invested in each portfolio. The low volatility quintile 1 portfolio delivered a compound return of 10.9% annually while the high volatility quintile 5 portfolio returned only 6.3% annually, resulting in an enormous difference in terminal wealth. Clearly, return volatility is a critical element of factor analysis, in this case revealing a useful stock selection factor that would be discarded by any researcher looking only at average returns across the rank spectrum.

Figure 1: Comparative Compounded Wealth

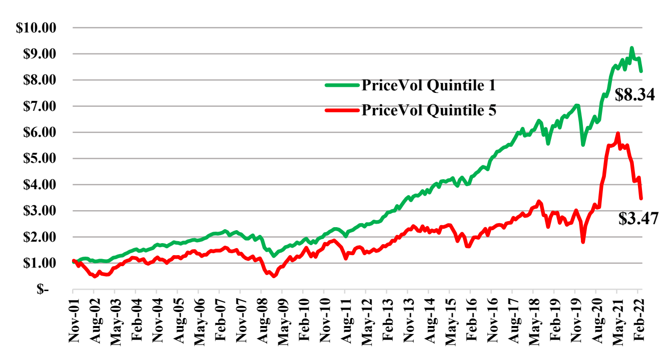

Table 4 shows the annualized compounded returns for each factor's quintile portfolios. Comparing various cohort statistics in Tables 3 and 4 provides further perspective into how return level and return volatility interact and their relative importance. For example, Table 3 shows that the Price Volatility quintile 2 portfolio had a slightly higher average monthly return than the quintile 1 portfolio (1.01% vs 0.94%), but a much larger return standard deviation (7.57% vs 5.85%). Table 4 shows that the small average return advantage of the quintile 2 portfolio outweighed its larger volatility disadvantage, resulting in a compound annual return of 11.2% vs 10.9% for the quintile 1 portfolio. The cohort portfolio with the highest average return (1.13%) in Table 3 – Beta quintile 3 – also had the highest compound return (12.4%) in Table 4, so the importance of higher average return is apparent in comparing factors/strategies.

Table 4: Annual Compound %Returns					
Factor	**Qn1**	**Qn2**	**Qn3**	**Qn4**	**Qn5**
PriceVol	10.9	11.2	10.3	8.4	6.3
Beta	9.5	11.6	12.4	11.5	6.1
Mktcap	9.8	10.1	9.6	10.4	8.2

But lower volatility is also important. For example, Beta quintile 4 had the same average return (1.13%) as Beta quintile 3 but provided a lower compound return (11.5% vs 12.4%) due to its higher volatility (10.31% vs 9.37%). Also note how Beta quintile 4 had a higher average return than the Beta quintile 2 (1.13% vs 1.03%), yet provided a lower compound return (11.5% vs 11.6%) due to its higher volatility (10.31% vs 8.77%).

Conclusion

Many academic and practitioner research studies focus on average factor group returns in determining the power of potential stock selection criteria. In this study, we have shown that return volatility is a critical element of stock selection factor analysis that can lead to entirely different conclusions about a factor's stock selection usefulness.

This is not just a statistical argument. For example, many investment products (e.g., ETFs) have been launched in recent years based on analysis that showed that certain stock groups (e.g., cheap valuation, high momentum) have historically had above-average returns. Many of these strategies have performed poorly because the stocks they hold also have above-average volatility which drags down their compound returns.

The good news is that RIR has found numerous factors correlated with both average returns *and* volatility (e.g., inputs to Downside Risk Alert), making them particularly useful in real-world portfolio investment strategies.